

## **Rayleigh-Taylor instability of a plasma in presence of a variable magnetic field and suspended particles in porous medium**

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**Abstract** : The Rayleigh-Taylor instability of a plasma in a porous medium is considered in the presence of a variable horizontal magnetic field and suspended particles. The case of two uniform fluids separated by a horizontal boundary and exponentially varying density, viscosity, magnetic field and particle number density are considered. In each case, the magnetic field succeeds in stabilizing waves in a certain wave-number range, which were unstable in the absence of magnetic field, whereas the system is found to be stable for potentially stable configuration/stable stratification. The growth rate both increase (for certain wave numbers) and decrease (for different wave numbers) with the increase in kinematic viscosity, medium permeability and particle number density. The medium permeability and suspended particles do not have any qualitative effect on the nature of stability or instability. This is in contrast to the thermal instability (Bénard convection) problem in porous medium, where the medium permeability and suspended particles have a destabilizing effect

**Keywords** : Rayleigh-Taylor instability, porous medium, suspended particles.

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### **1. Introduction**

The stability derived from the character of the equilibrium of an incompressible heavy fluid of variable density (*i.e.* of a heterogeneous fluid) is termed as Rayleigh-Taylor instability. Mention may be made of two important special cases : (a) two fluids of different densities superposed one over the other; (b) a fluid with a continuous density stratification. The instability of the plane interface separating two fluids when one is accelerated towards the

other or when one is superposed over the other, has been studied by several authors, and Chandrasekhar [1] has given a detailed account of these investigations in non-porous medium.

Generally, it is accepted that comets consist of a dusty 'snowball' of a mixture of frozen gases which in the process of their journey, changes from solid to gas and *vice versa*. The physical properties of comets, meteorities and interplanetary dust strongly suggest the importance of porosity in astrophysical context (McDonnell [2]). In stellar interiors and atmospheres, the magnetic field may be (and quite often is) variable (and non-uniform) and may altogether alter the nature of the instability. When a fluid permeates a porous material, the gross effect is represented by the Darcy's law. As a result of this macroscopic law, the usual viscous term in the equations of fluid motion is replaced by the resistance term  $(-\mu/k_1)u$  where  $\mu$  is the viscosity of the fluid,  $k_1$  is the permeability of the medium and  $u$  is the Darcian (filter) velocity of the fluid, calculated from Darcy's law. Wooding [3] has considered the Rayleigh instability of a thermal boundary layer in flow through a porous medium.

Recent spacecraft observations have confirmed that dust particles play an important role in the dynamics of the Martian atmosphere as well as in the diurnal and surface variations in the temperature of the Martian weather. It is therefore, of interest to study the presence of suspended (fine dust) particles in astrophysical situations. In geophysical situations, more often than not, the fluid is not pure but may instead be permeated with suspended (or dust) particles. Scanlon and Segel [4] studied the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by that of the particles. Sharma *et al* [5] considered the effect of suspended particles on the onset of Bénard convection in hydromagnetics. The thermal instability of fluids through a porous medium in the presence of suspended particles has been studied by Sharma and Sharma [6]. The effect of suspended particles and medium permeability were found to destabilize the layer. Sharma and Sunil [7] have studied the Rayleigh-Taylor instability of a partially ionized plasma in a porous medium, in the presence of a variable magnetic field. In another study, Sharma and Sunil [8] have considered the thermal instability of compressible Hall plasma in the presence of suspended particles. Recently, Sharma and Sunil [9] have studied the thermal instability of Oldroydian viscoelastic fluid with suspended particles in hydromagnetics in porous medium.

The effect of suspended particles on the stability of superposed fluids might be of industrial and chemical engineering importance. Further motivation for this study is the fact that knowledge concerning fluid-particle mixtures is not commensurate with their industrial and scientific importance. The present paper is devoted to the consideration of Rayleigh-Taylor instability of a plasma in the presence of a variable horizontal magnetic field and suspended particles in porous medium.

## 2. Basic equations

Consider a static state in which an incompressible plasma particle layer of variable density is arranged in horizontal strata and the pressure  $p$  and the density  $\rho$  are functions of the vertical coordinate  $z$  only. The character of the equilibrium of this initial static state is determined by supposing that the system is slightly disturbed and then following its further evolution. The fluid is under the action of gravity  $\mathbf{g}$  ( $0, 0, -g$ ) and the variable horizontal magnetic field  $\mathbf{H}$  ( $H_0(z), 0, 0$ ). The particles are assumed to be non-conducting. This plasma particle layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity  $\epsilon$  and medium permeability  $k_1$ .

Let  $\rho, \mu, p$  and  $\mathbf{u}$  ( $u, v, w$ ) denote respectively, the density, the viscosity, the pressure and the velocity of the hydromagnetic plasma,  $\mathbf{V}(\bar{x}, t)$  and  $N(\bar{x}, t)$  denote the velocity and number density of the particles, respectively.  $K = 6\pi\mu\eta$ , where  $\eta$  is the particle radius, is a constant and  $\bar{x} = (x, y, z)$ . Then the equations of motion and continuity for the hydromagnetic plasma are

$$\frac{\rho}{\epsilon} \left[ \frac{\partial \mathbf{u}}{\partial t} + \frac{1}{\epsilon} (\mathbf{u} \cdot \nabla) \mathbf{u} \right] = -\nabla p + \rho \mathbf{g} + \frac{\mu}{4\pi} (\nabla \times \mathbf{H}) \times \mathbf{H} - \frac{\mu}{k_1} \mathbf{u} + \frac{KN}{\epsilon} (\mathbf{V} - \mathbf{u}), \quad (1)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (2)$$

where  $\mu_c$ , the magnetic permeability, is assumed to be constant and the plasma is assumed to be infinitely conducting.

Since the density of a particle moving with the plasma remains unchanged, we have

$$\epsilon \frac{\partial \rho}{\partial t} + (\mathbf{u} \cdot \nabla) \rho = 0. \quad (3)$$

The Maxwell's equations give

$$\epsilon \frac{\partial \mathbf{H}}{\partial t} = (\mathbf{H} \cdot \nabla) \mathbf{u} - (\mathbf{u} \cdot \nabla) \mathbf{H}, \quad (4)$$

$$\nabla \cdot \mathbf{H} = 0. \quad (5)$$

The presence of particles adds an extra force term, proportional to the velocity difference between particles and plasma and appears in equations of motion (1). Since the force exerted by the plasma on the particles is equal and opposite to that exerted by the particles on the plasma, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles. The buoyancy force on the particles is neglected. Interparticle reactions are not considered for we assume that the distance between particles is quite large compared with their diameter.

The equations of motion and continuity for the particles, under the above approximations, are

$$mN \left[ \frac{\partial \mathbf{V}}{\partial t} + \frac{1}{\varepsilon} (\mathbf{V} \cdot \nabla) \mathbf{V} \right] = KN(\mathbf{u} - \mathbf{V}), \quad (6)$$

$$\varepsilon \frac{\partial N}{\partial t} + \nabla \cdot (N\mathbf{V}) = 0, \quad (7)$$

where  $mN$  is the mass of the particles per unit volume.

Let  $\delta\rho$ ,  $\delta p$ ,  $\mathbf{u}$  ( $u, v, w$ ),  $\mathbf{V}$  ( $l, r, s$ ) and  $\mathbf{h}$  ( $h_1, h_v, h_z$ ) denote, respectively, the perturbations in the hydromagnetic plasma density  $\rho(z)$ , pressure  $p(z)$ , plasma velocity  $\mathbf{u}$  ( $O, O, O$ ), particle velocity  $\mathbf{V}$  ( $O, O, O$ ) and the magnetic field  $\mathbf{H}$  ( $H(z), O, O$ ). Then, the linearized hydromagnetic perturbation equations governing the motion of the plasma particle layer through porous medium, are

$$\begin{aligned} \frac{\rho}{\varepsilon} \frac{\partial \mathbf{u}}{\partial t} = & -\nabla \delta p + \mathbf{g} \delta \rho + \frac{\mu_c}{4\pi} [(\nabla \times \mathbf{h}) \times \mathbf{H} + (\nabla \times \mathbf{H}) \times \mathbf{h}] \\ & - \frac{\mu}{k_1} \mathbf{u} + \frac{KN}{\varepsilon} (\mathbf{V} - \mathbf{u}), \end{aligned} \quad (8)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (9)$$

$$\varepsilon \frac{\partial \mathbf{h}}{\partial t} = (\mathbf{H} \cdot \nabla) \mathbf{u} - (\mathbf{u} \cdot \nabla) \mathbf{H}, \quad (10)$$

$$\nabla \cdot \mathbf{h} = 0 \quad (11)$$

$$\left[ \frac{m}{K} \frac{\partial}{\partial t} + 1 \right] \mathbf{V} = \mathbf{u}. \quad (12)$$

In addition to eqs. (8)–(12), we have the equation

$$\varepsilon \frac{\partial}{\partial t} \delta \rho = -\omega \left( \frac{d\rho}{dz} \right), \quad (13)$$

which ensures that the density of every particle remains unchanged as we follow it with its motion.

### 3. Dispersion relation

Analysing the disturbances into normal modes, we seek solutions whose dependence on  $x, y$  and  $t$  is given by

$$\exp(ik_x x + ik_y y + nt), \quad (14)$$

where  $k_x, k_y$  are the horizontal components of the wave number,  $k = (k_x^2 + k_y^2)^{1/2}$  is the resultant wave number, and  $n$  is the growth rate which is, in general, a complex constant.

Eliminating  $V$  between (8) and (12) and using (14), (8)–(13) gives

$$\left[ n' + \frac{v}{k_1} \right] \rho u = -ik_x \delta p + \frac{\mu_e}{4\pi} h_z (DH_0), \quad (15)$$

$$\left[ n' + \frac{v}{k_1} \right] \rho v = -ik_y \delta p + \frac{\mu_e H_0}{4\pi} (ik_x h_y - ik_y h_x), \quad (16)$$

$$\left[ n' + \frac{v}{k_1} \right] \rho w = -D\delta p - g\delta\rho + \frac{\mu_e H_0}{4\pi} \left( ik_x h_z - Dh_x - h_x \frac{DH_0}{H_0} \right), \quad (17)$$

$$ik_x u + ik_y v + Dw = 0, \quad (18)$$

$$ik_x h_x + ik_y h_y + Dh_z = 0, \quad (19)$$

$$\epsilon n h_x = ik_x H_0 u - w(DH_0), \quad (20)$$

$$\epsilon n h_y = ik_y H_0 v, \quad (21)$$

$$\epsilon n h_z = ik_x H_0 w, \quad (22)$$

$$\epsilon n \delta p = -w(D\rho), \quad (23)$$

where  $n' = \frac{n}{\epsilon} \left[ 1 + \frac{mNK}{mn + K} \right]$ ,  $v = \frac{\mu}{\rho}$ ,

and  $D = \frac{d}{dz}$ .

Eq. (16) with the help of (20) and (21) becomes

$$\left[ n' + \frac{v}{k_1} \right] \rho v = -ik_y \delta p + \frac{\mu_e H_0}{4\pi \epsilon n} (ik_x H_0 \zeta + ik_y w DH_0), \quad (24)$$

where  $\zeta = ik_x v - ik_y u$  is the  $z$ -component of vorticity.

Multiplying (15) by  $-ik_x$ , (24) by  $-ik_y$ , adding and using (18), we obtain

$$\begin{aligned} \left[ n' + \frac{v}{k_1} \right] \rho Dw = & -k^2 \delta p + \frac{\mu_e k_x k_y H_0^2}{4\pi \epsilon n} \zeta + \frac{\mu_e k_y^2 H_0}{4\pi \epsilon n} (DH_0) w \\ & - \frac{i\mu_e k_x}{4\pi} h_z (DH_0). \end{aligned} \quad (25)$$

Eliminating  $\delta p$  between (17) and (25) and using (18)–(23), we get

$$\begin{aligned} n' [D(\rho Dw) - k^2 \rho w] + \frac{1}{k_1} [D(\rho v Dw) - k^2 \rho vw] + \frac{\mu_e H_0^2 k^2}{4\pi \epsilon n} (D^2 - k^2) w \\ + \frac{\mu_e k_x^2}{4\pi \epsilon n} D(H_0^2) Dw + \frac{gk^2}{\epsilon n} (D\rho) w = 0. \end{aligned} \quad (26)$$

#### 4. Two uniform plasmas separated by a horizontal boundary

Consider the case when two superposed plasmas of uniform densities  $\rho_1$  and  $\rho_2$ , uniform viscosities  $\mu_1$  and  $\mu_2$  and uniform magnetic fields  $H_{01}$  and  $H_{02}$  are separated by a horizontal boundary at  $z = 0$ . The subscripts 1 and 2 distinguish the lower and the upper plasmas respectively. Then, in each region of constant  $\rho$ ,  $\mu$  and  $H$ , eq. (26) reduces to

$$(D^2 - k^2) w = 0. \quad (27)$$

The general solution of (27) is

$$w = Ae^{+kz} + Be^{-kz}, \quad (28)$$

where  $A$  and  $B$  are arbitrary constants.

The boundary conditions to be satisfied here are :

- (i) The velocity  $w$  should vanish when  $z \rightarrow -\infty$  (for the lower plasmas) and  $z \rightarrow +\infty$  (for the upper plasmas)
- (ii)  $w(z)$  is continuous at  $z = 0$ .
- (iii) The pressure should be continuous across the interface.

Applying the boundary conditions (i) and (ii), we have

$$w_1 = Ae^{+kz}, \quad (z < 0), \quad (29)$$

$$w_2 = Ae^{-kz}, \quad (z > 0), \quad (30)$$

The same constant  $A$  being chosen to ensure the continuity at  $z = 0$

The continuity of pressure implies that

$$n' \Delta_0(\rho Dw) + \frac{1}{k_1} \Delta_0(\rho v Dw) + \frac{\mu_e k_1^2}{4\pi\epsilon n} \Delta_0(H_0^2 Dw) + \frac{gk^2}{\epsilon n} \Delta_0(\rho) w_0 = 0. \quad (31)$$

Applying the condition (31) to the solutions (29) and (30), we obtain

$$\begin{aligned} n^3 + \left[ \frac{K}{m} \left( 1 + \frac{mN}{\rho} \right) + \frac{\epsilon}{k_1} (\alpha_1 v_1 + \alpha_2 v_2) \right] n^2 + \left[ \frac{\epsilon}{k_1} \frac{K}{m} (\alpha_1 v_1 + \alpha_2 v_2) \right. \\ \left. + \{ 2k_1^2 V_A^2 - gk(\alpha_2 - \alpha_1) \} \right] n + \frac{K}{m} [ 2k_1^2 V_A^2 - gk(\alpha_2 - \alpha_1) ] = 0, \end{aligned} \quad (32)$$

where 
$$\alpha_{1,2} = \frac{\rho_{1,2}}{\rho_1 + \rho_2}, \quad v_{1,2} = \frac{\mu_{1,2}}{\rho_{1,2}}.$$

For the sake of simplicity, we assume that the Alfvén velocities of the two plasmas are the same, so that

$$V_A^2 = \frac{\mu_e H_{01}^2}{4\pi(\rho_1 + \rho_2)} = \frac{\mu_e H_{02}^2}{4\pi(\rho_1 + \rho_2)}. \quad (33)$$

(ii) *Stable case* ( $\rho_1 > \rho_2$ )

For the potentially stable case,  $\alpha_1 > \alpha_2$  and eq. (32) does not involve any change of sign and so does not allow any positive root. The system is therefore, stable.

(b) *Unstable case* ( $\rho_2 > \rho_1$ )

For the potentially unstable case, if

$$2k_x^2 V_A^2 > gk(\alpha_2 - \alpha_1), \quad (34)$$

(32) does not admit any change of sign and so has no positive root. Therefore, the system is stable.

$$\text{If } 2k_x^2 V_A^2 < gk(\alpha_2 - \alpha_1), \quad (35)$$

The constant term in (32) is negative. Eq. (32) therefore, allows one change of sign and so has one positive root. The occurrence of a positive root implies that the system is unstable.

Thus for the unstable case ( $\rho_2 > \rho_1$ ), the system is stable or unstable according as  $\frac{\mu H^2 k_x^2}{\pi}$  is greater than or smaller than  $gk(\rho_2 - \rho_1)$ . In the absence of a magnetic field, (32) has one positive root, and so the system is unstable for  $\rho_2 > \rho_1$ . But the magnetic field has got a stabilizing effect and completely stabilizes the wave-number band  $k > k^*$ , where

$$k^* = \frac{2\pi g(\rho_2 - \rho_1)}{\mu_e H_0^2} \sec^2 \theta, \quad (36)$$

and  $\theta$  is the inclination of the wave vector  $k$  to the direction of the magnetic field  $H$  i.e.  $k_x = k \cos \theta$ .

### 5. The case of exponentially varying density, viscosity, magnetic field and particles number density.

Eq (26) can be written as

$$\begin{aligned} n' \left[ D(\rho Dw) - k^2 \rho w \right] + \frac{1}{k_1} \left[ D(\rho v Dw) - k^2 \rho v w \right] + \frac{gk^2}{\epsilon n} (D\rho)w \\ + \frac{\mu_e k_x^2}{4\pi \epsilon n} D(H_0^2 Dw) - \frac{\mu_e k_x^2 H_0^2 k^2}{4\pi \epsilon n} w = 0, \end{aligned} \quad (37)$$

Let us assume

$$\left. \begin{aligned} \rho &= \rho_0 e^{\beta z}, \quad N = N_0 e^{\beta z}, \\ \mu &= \mu_0 e^{\beta z}, \quad H_0^2(z) = H_1^2 e^{\beta z}, \end{aligned} \right\} \quad (38)$$

where  $\rho_0, N_0, \mu_0, H_1$  and  $\beta$  are constants. Eqs. (38) imply that the coefficient of kinematic viscosity  $\nu$  and the Alfvén velocity  $V_A$  are constants everywhere.

Substituting the values of  $\rho$ ,  $N$ ,  $\mu$ ,  $H_0^2$  in (37) and neglecting the effect of heterogeneity on inertia, we obtain

$$\left[ \frac{n'}{v_0} + \frac{1}{k_1} + \frac{k_x^2 V_A^2}{v_0 \epsilon n} \right] (D^2 - k^2) w + \frac{g \beta k^2}{v_0 \epsilon n} = 0, \quad (39)$$

where  $v_0 = \frac{\mu_0}{\rho_0}$ .

Consider the case of two free boundaries. The boundary conditions for the case of two free surfaces are

$$\omega = D^2 \omega = 0 \text{ at } z = 0 \text{ and } z = d. \quad (40)$$

The proper solution of (39) satisfying (40) is

$$w = A \sin \frac{m\pi z}{d}, \quad (41)$$

where  $A$  is a constant and  $m$  is any integer.

Substituting (41) in (39), we obtain the dispersion relation

$$\left[ \frac{n'}{v_0} + \frac{1}{k_1} + \frac{k_x^2 V_A^2}{v_0 \epsilon n} \right] \left[ \left( \frac{m\pi}{d} \right)^2 + k^2 \right] - \frac{g \beta k^2}{v_0 \epsilon n} = 0. \quad (42)$$

Letting  $(\frac{m\pi}{d})^2 + k^2 = L$ , the above equation on simplification, becomes

$$n^3 + \left[ \frac{K}{m} \left( 1 + \frac{m N_0}{\rho_0} \right) + \frac{\epsilon v_0}{k_1} \right] n^2 + \left[ \frac{\epsilon v_0}{k_1} \frac{K}{m} + \left( k_x^2 V_A^2 - \frac{g \beta k^2}{L} \right) \right] n + \frac{K}{m} \left( k_x^2 V_A^2 - \frac{g \beta k^2}{L} \right) = 0. \quad (43)$$

(a) *Stable stratification* ( $\beta < 0$ )

For the stable stratification ( $\beta < 0$ ), Eq. (43) does not admit any positive root of  $n$  and so the system is always stable for disturbances of all wave numbers.

(b) *Unstable stratification* ( $\beta > 0$ )

For the unstable stratification ( $\beta > 0$ ), the system is stable or unstable according as

$$k_x^2 V_A^2 > \text{ or } < \frac{g \beta k^2}{L}. \quad (44)$$

In the absence of magnetic field, the system is clearly unstable for  $\beta > 0$ . However, the system can be completely stabilized by a magnetic field as can be seen from (44), if

$$V_A^2 > \frac{g \beta k^2}{k_x^2 L} \quad (45)$$



Therefore, the magnetic field succeeds in stabilizing wave numbers in the range

$$k^2 > \frac{g\beta}{V_A^2} \sec^2 \theta - \left( \frac{m\pi}{d} \right)^2, \quad (46)$$

which were unstable in the absence of a magnetic field.

The medium permeability and suspended particles do not have any qualitative effect on the nature of the stability or instability. This is in contrast to the thermal instability Bénard convection in porous medium where the medium permeability and suspended particles have destabilizing effect. Thus, if

$$\beta > 0 \text{ and } k_x^2 V_A^2 < \frac{g\beta k^2}{L}, \quad (47)$$

Eq (43) has one positive root. Let  $n_0$  denote the positive root of (43).

$$\begin{aligned} \text{Then } n_0^3 + \left[ \frac{K}{m} \left( 1 + \frac{mN_0}{\rho_0} \right) + \frac{\varepsilon v_0}{k_1} \right] n_0^2 + \left[ \frac{\varepsilon v_0}{k_1} \frac{K}{m} + \left( k_x^2 V_A^2 - \frac{g\beta k^2}{L} \right) \right] n_0 \\ + \frac{K}{m} \left( k_x^2 V_A^2 - \frac{g\beta k^2}{L} \right) = 0 \end{aligned} \quad (48)$$

To study the behaviour of growth rate of unstable modes with respect to viscosity, medium permeability and particle number density, we examine the natures of  $\frac{dn_0}{dv_0}$ ,  $\frac{dn_0}{dk_1}$  and  $\frac{dn_0}{dN_0}$  analytically. Eq. (48) yields

$$\frac{dn_0}{dv_0} = - \frac{\frac{\varepsilon}{k_1} n_0 \left( n_0 + \frac{K}{m} \right)}{\left[ k_x^2 V_A^2 - \frac{g\beta k^2}{L} \right] + \left[ 3n_0^2 + 2n_0 \left\{ \frac{K}{m} \left( 1 + \frac{mN_0}{\rho_0} \right) + \frac{\varepsilon v_0}{k_1} \right\} + \frac{\varepsilon v_0}{k_1} \frac{K}{m} \right]}. \quad (49)$$

$$\frac{dn_0}{dk_1} = \frac{\frac{\varepsilon v_0}{k_1^2} n_0 \left( n_0 + \frac{K}{m} \right)}{\left[ k_x^2 V_A^2 - \frac{g\beta k^2}{L} \right] + \left[ 3n_0^2 + 2n_0 \left\{ \frac{K}{m} \left( 1 + \frac{mN_0}{\rho_0} \right) + \frac{\varepsilon v_0}{k_1} \right\} + \frac{\varepsilon v_0}{k_1} \frac{K}{m} \right]}. \quad (50)$$

$$\frac{dn_0}{dN_0} = - \frac{\left( \frac{K}{\rho_0} \right) n_0^2}{\left[ k_x^2 V_A^2 - \frac{g\beta k^2}{L} \right] + \left[ 3n_0^2 + 2n_0 \left\{ \frac{K}{m} \left( 1 + \frac{mN_0}{\rho_0} \right) + \frac{\varepsilon v_0}{k_1} \right\} + \frac{\varepsilon v_0}{k_1} \frac{K}{m} \right]}. \quad (51)$$

It is evident from (49) and (51) that if, in addition to  $k^2 > \frac{k^2 V_A^2 L}{g\beta}$ , which is a sufficient condition for instability, we have the condition

$$\left| k_i^2 V_A^2 - \frac{g\beta k^2}{L} \right| < \left[ 3n_0^2 + 2n_0 \left\{ \frac{K}{m} \left( 1 + \frac{mN_0}{\rho_0} \right) + \frac{\epsilon V_0}{k_i} \right\} + \frac{\epsilon V_0}{k_i} \frac{K}{m} \right]. \quad (52)$$

$\frac{dn_0}{dV_0}$  and  $\frac{dn_0}{dN_0}$  are always negative. The growth rates therefore, decrease with increase in viscosity and particle number density. However, the growth rates increase with increase in viscosity and particle number density, if

$$\left| k_i^2 V_A^2 - \frac{g\beta k^2}{L} \right| > \left[ 3n_0^2 + 2n_0 \left\{ \frac{K}{m} \left( 1 + \frac{mN_0}{\rho_0} \right) + \frac{\epsilon V_0}{k_i} \right\} + \frac{\epsilon V_0}{k_i} \frac{K}{m} \right], \quad (53)$$

for then  $\frac{dn_0}{dV_0}$  and  $\frac{dn_0}{dN_0}$  are positive.

Similarly if the inequality (52) holds good, then  $\frac{dn_0}{dk_i}$  is positive. Therefore, the growth rates increase with increase in medium permeability. However, if the inequality (53) holds good, then  $\frac{dn_0}{dk_i}$  is negative then, the growth rate decrease with increase in medium permeability.

## 6. Conclusion

In geophysical situations, more often than not, the fluid is not pure but may instead be permeated with suspended (or dust) particles. Recently, dusty plasma have become the subject of intensive investigations [10–14]. Now a days, it is well recognised that the dust particles suspended in a plasma acquire a significant amount of charge and this brings a dramatic change in the overall dynamics of the system. Charged suspended particles in a plasma often exist in the Universe and there are several situations in which the interaction between the charged dust and plasma components becomes important in cosmic physics, astrophysics and industrial plasma. Here, we have considered only heat capacity of the suspended particles/dust without taking into consideration of charge on it. Also we assume that the distances between the particles are quite large compared with their diameter so that the interparticle reactions are ignored. The effect of pressure, gravity and Darcian force on the particles are negligibly small and therefore, ignored. Strömgen [15] has reported that ionized hydrogen is limited to certain rather sharply bounded regions in space surroundings, for examples, O-type stars and clusters of such stars, and that the gas with dust particles outside these regions is essentially non-ionized. Other examples of such situations are given by Alfvén's [16] theory of the origin of the planetary system, where a high ionization rate is suggested to appear from collisions between a plasma and a neutral dust cloud and by the absorption of plasma waves due to ion-neutral collisions, such as, in the solar photosphere and chromosphere and in cool interstellar clouds [17,18].

The present paper attempts to study the Rayleigh-Taylor instability of a plasma in presence of a variable magnetic field and suspended/dust particles in a porous medium. We

conclude the whole analysis with the following statements. Here, the cases of two uniform plasma separated by a horizontal boundary and exponentially varying density, viscosity, magnetic field and particle number density are considered; and in each case, the magnetic field succeeds in stabilizing waves in a certain wave number range which were unstable in the absence of magnetic field, whereas the system is found to be stable for stable configuration/stable stratification. The medium permeability and suspended particles do not have any qualitative effect on the nature of the stability or instability. This is in contrast to the thermal instability (Bénard convection) problem in porous medium, where the suspended particles and medium permeability have a destabilizing effect. The growth rate both increase (for certain wave numbers) and decrease (for different wave numbers) with increase in viscosity, medium permeability and particle number density.

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### References

- [1] S Chandrasekhar *Hydrodynamic and Hydromagnetic Stability* (New York : Dover) (1981)
- [2] J A M McDonnell *Cosmic Dust* (Toronto : John Wiley and Sons) (1978)
- [3] R A Wooding *J. Fluid. Mech.* **9** 183 (1960)
- [4] J W Scanlon and L A Segel *Phys. Fluids* **16** 1573 (1973)
- [5] R C Sharma, K Prakesh and S N Dube *Acta Phys. Hung.* **40** 3 (1976)
- [6] R C Sharma and K N Sharma *J. Math. Phys. Sci.* **16** 167 (1982)
- [7] R C Sharma and Sunil *Z. Naturforsch.* **47a** 1227 (1992)
- [8] R C Sharma and Sunil *Ganita* **44(1)** 1 (1983)
- [9] R C Sharma and Sunil *Polym -Plast. Technol. Engg.* **33(3)** 323 (1994)
- [10] C K Goertz *Rev. Geophys.* **27** 271 (1989)
- [11] C K Goertz *Adv. Space Res.* **4** 137 (1984)
- [12] U De Angelis, V Formisano and J Giordano *J. Plasma Phys.* **40** 399 (1988)
- [13] UA Mofiz M Islam and Z Ahmed *J. Plasma Phys.* **50(1)** 37 (1993)
- [14] A Sen and P K Kaw (eds) *A Variety of Plasma* (Oxford : Oxford University Press) (1989)
- [15] B Strömgren *Astrophys. J.* **89** 526 (1939)
- [16] H Alfvén *The Origin of the Solar System* (Oxford : Clarendon) (1954)
- [17] J H Piddington *Mon. Noti. Roy. Astron. Soc.* **14** 638 (1954)
- [18] B Lehnert *Suppl. Nuovo Cim.* **13** 59 (1959)